# Crack Driving Force Expressions Using Compliance Approach in Clamped Beam Bending Geometry

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## 1. Introduction

In the present research, numerical simulations were carried through the extended finite element method (XFEM) to derive crack driving force expressions for different beam aspect ratios (L/W) using an energy-based compliance approach. The crack driving force in linear elastic fracture mechanics (LEFM), which is also referred to as the energy release rate (G), from the compliance approach was compared with the J-integral approach in an isotropic linear elastic brittle material polymethyl methacrylate (PMMA). Experimental validation of the expressions was carried through the evaluation of fracture toughness as critical energy release rate ($G_c$) of the material.

## 2. Clamped Beam Geometry

The edge-notched clamped beam geometry (Figure 1) provides the geometric stability for controlled crack growth, as shown in the previous work[1][2]. This opens up prodigious possibilities in fracture toughness testing and understanding toughening mechanisms in a wide range of materials. Mesh convergence study shows 0.3 mm local mesh size to be optimal (Figure 2)

## 3. Energy Release Rate (G)

The compliance approach is a convenient approach to execute in a realistic scenario. G is obtained as a function of compliance change as the crack advances (Figure 3 (a)). J-integral, which is equivalent to G for linear elastic materials, was also directly evaluated through the XFEM, based on the contour convergence study (Figure 3 (b)). For this, the J-integral data was extracted from the output data files of Abaqus CAE v6.14 - 4 model and analyzed using Python.

**Figure 1:** Clamped beam bending geometry

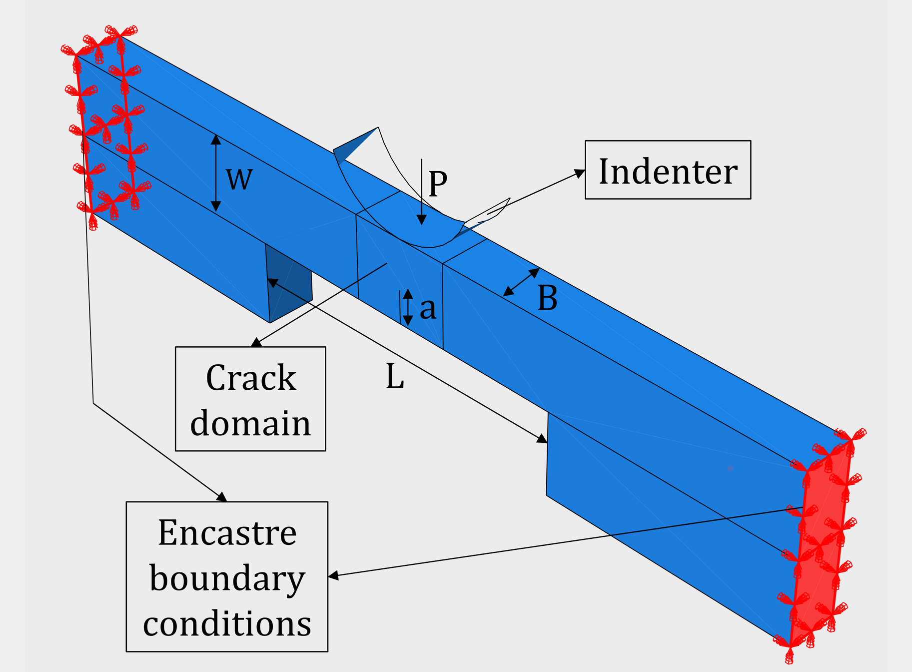



**Figure 2:** Mesh convergence study

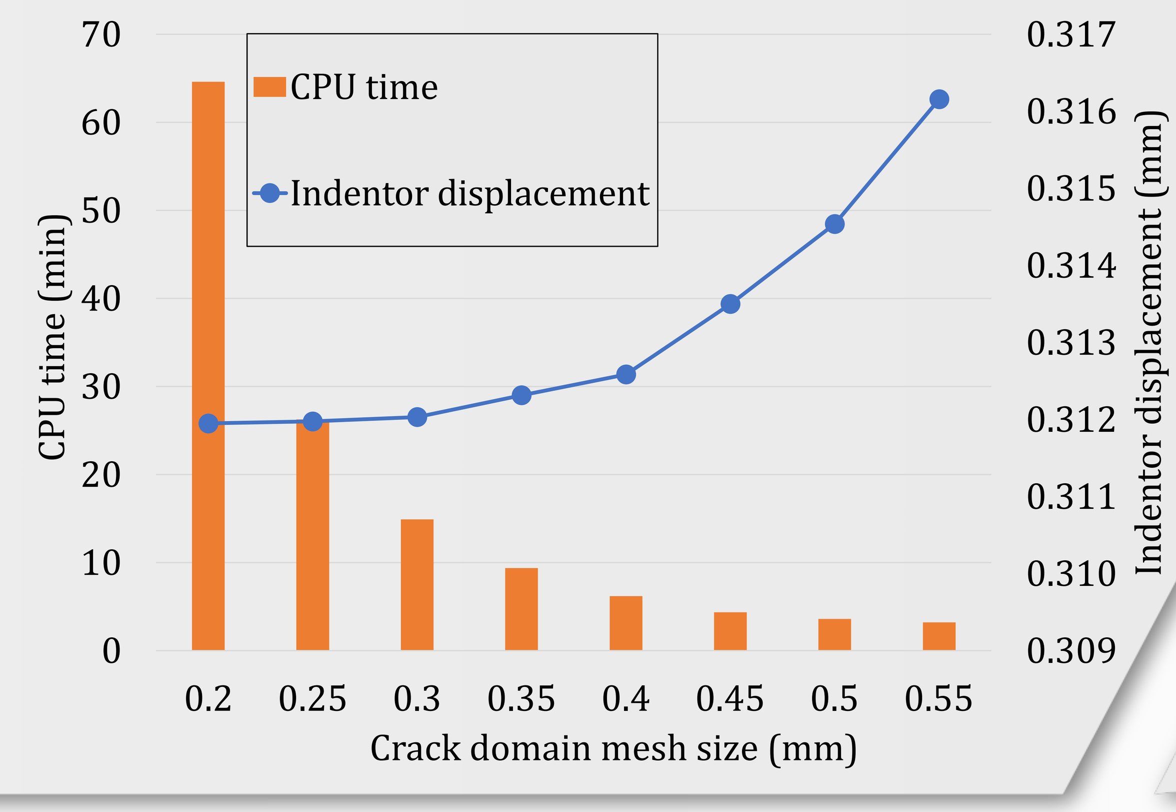



**Figure 3**: (a) Compliance approach
(b) Contours for J-integral evaluation

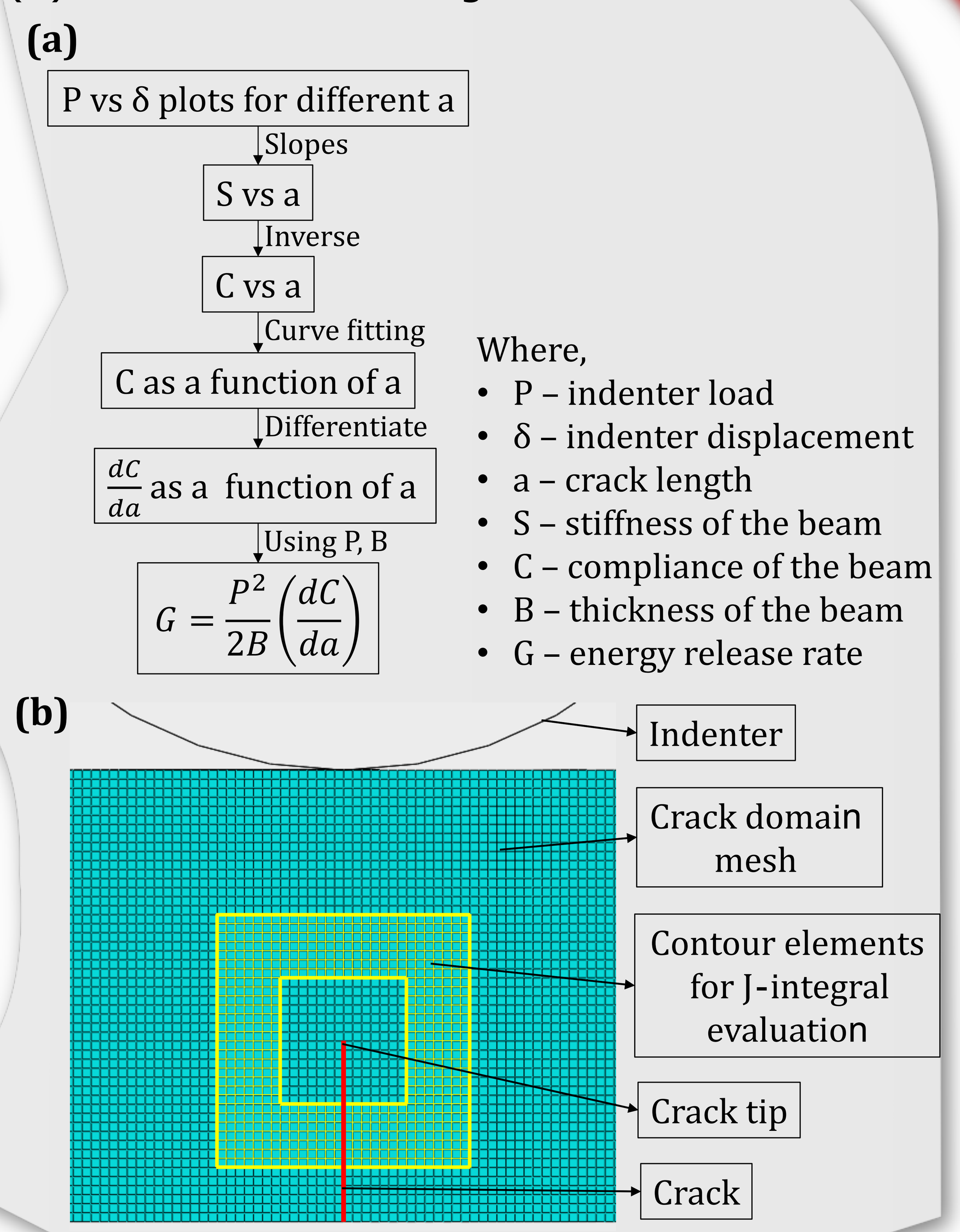
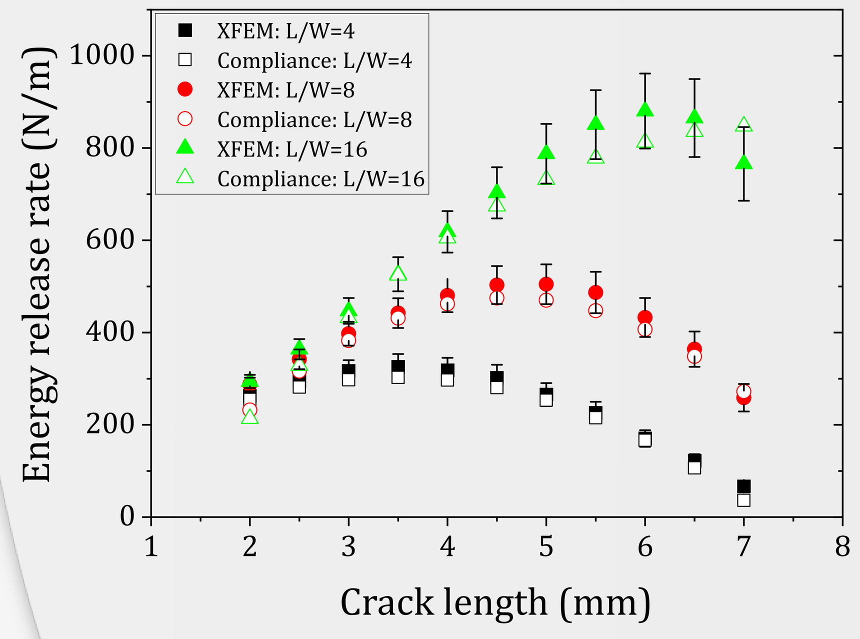
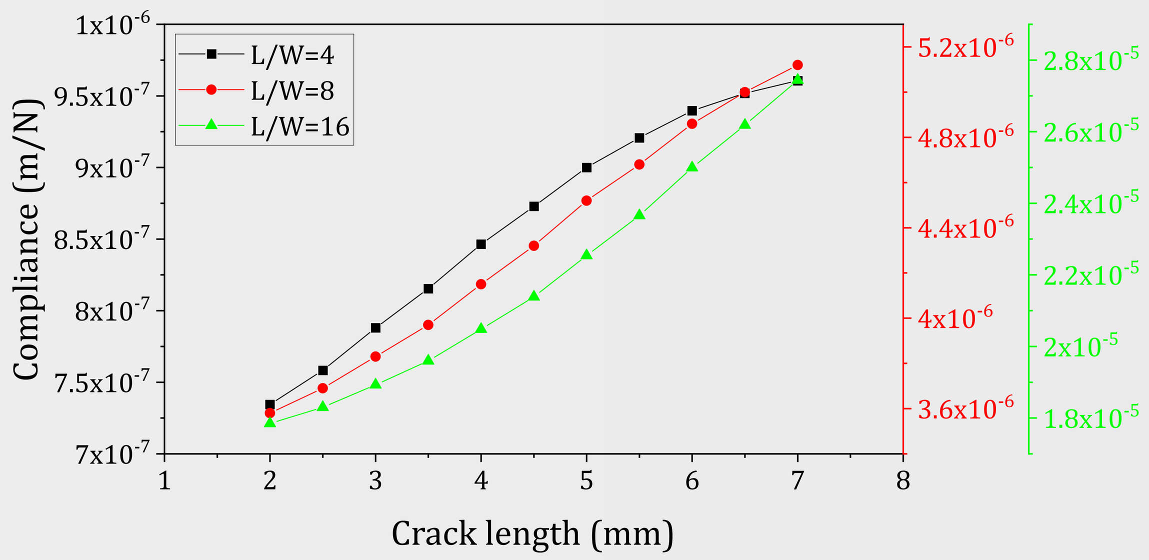



**Figure 5:** G vs a from XFEM (as J-integral) and compliance approach for different L/W ratios




$$G = \frac{P^2}{2B}\left(3B_3a^2 + 2B_2a + B_1\right) \quad (2)$$

**Figure 4:** C vs a for different L/W ratios




$$C = B_3a^3 + B_2a^2 + B_1a + B_0 \quad (1)$$

Where $B_3, B_2, B_1$ and $B_0$ are the fitting coefficients

**Table 1:** Experimental validation of fracture toughness

| L/W | a | $P_c$ | $G_c$ | $K_{Ic}\left[=\sqrt{G(1-\mu^2)}\right]$ | Reference $K_{Ic}$[3] |
|---|---|---|---|---|---|
| | (mm) | N | N/m | MPa - $m^{1/2}$ | MPa -$m^{1/2}$ |
| 4 | 4.8 | 177 | 145.2 | 0.77 | 0.75 ± 0.02 |
| 6 | 5.3 | 96 | 119.7 | 0.70 | 0.72 ± 0.01 |
| 8 | 6.3 | 65 | 103.2 | 0.65 | 0.70 ± 0.01 |

## 4. Fracture Toughness Evaluation

The fracture toughness as $G_c$ and mode-I critical stress intensity factor ($K_{Ic}$) were evaluated using experimentally determined critical failure load ($P_c$) and the expressions obtained for G (Table 1).

## 5. Results & Discussions

Compliance of the beam increases with crack length as well as L/W. These have been fit to a polynomial function (Equation 1) to obtain dC/da and hence G (Figure 4). Figure 5 demonstrates that the shorter beams attain the geometric stability at smaller crack lengths and hence the availability of the operating range of crack lengths to measure fracture behaviour such as the R-curve is higher. G obtained from the compliance show significant agreement with that from the J-integral approach. The energy-based approach have several advantages over the stress-based approach of K, especially in multiphase materials.

## 6. Conclusions

- Expressions of G were obtained and experimentally verified in clamped beams for homogeneous brittle material PMMA.
- In future these expressions will be used in evaluating mixed mode fracture and for predicting fracture behaviour of composite materials.

## References

[1] B. N. Jaya, V. Jayaram, S.K. Biswas, "A new method for fracture toughness determination of graded (Pt, Ni) Al bond coats by microbeam bend tests", Philos. Mag. 92 (25–27) (2012) 3326–3345.
[2] B. N. Jaya and V. Jayaram, "Crack stability in edge-notched clamped beam specimens: Modeling and experiments", Int. J. Fract., vol. 188, p. 213, 2014, doi: 10.1007/s10704-014-9956-2.
[3] A. Mishra, A. Lambai, V. Jayaram, and B. N. Jaya, "The edge-notched clamped beam bend specimen as a fracture toughness test geometry", Theor. Appl. Fract. Mech., vol. 105, p. 102409, 2019, doi: 10.1016/j.tafmec.2019.102409.

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